The ancient Greek riddle that helps us understand modern disease threats

May 2 2014, by Adam Kucharski

Finding an Achilles' heel. Credit: Γιάννης Ζήσης

Even in the face of death, Zeno of Elea knew how to frustrate people. Arrested for plotting against the tyrant Demylus, the ancient Greek philosopher refused to co-operate. The story goes that, rather than talk, he bit off his own tongue and spat it at his captor.

Zeno spent his life exasperating others. Prior to his demise, he had a reputation for creating baffling puzzles. He conjured up a series of apparently contradictory situations known as Zeno's Paradoxes, which have inspired centuries of debate among philosophers and mathematicians. Now the ideas are helping researchers tackle a far more dangerous problem.

Never-ending race

The most famous of Zeno's riddles is "Achilles and the tortoise". Trojan war hero Achilles lines up for a long-distance race against a tortoise (who presumably is still gloating after beating Aesop's hare). In the interests of fairness, Achilles gives the tortoise a head start – let's say of one mile. When the race starts, Achilles soon reaches the tortoise's starting position. However, in the time it takes him to arrive at this point, the tortoise has lumbered forward, perhaps by one tenth of a mile. Achilles quickly covers this ground, but the tortoise has again moved on.

Zeno argued that because the tortoise is always ahead by the time Achilles arrives at its previous position, the hero will never catch up. While the total distance Achilles has to run decreases each time, there are an infinite number of gaps to cover:

$1 + 1/10 + 1/100 + 1/1000 + \ldots$

And according to Zeno, "It is impossible to traverse an infinite number of things in a finite time."

It wasn't until the 19th Century that mathematicians proved Zeno wrong. As the distance between Achilles and the tortoise gets smaller and smaller, Achilles makes up ground faster and faster. In fact, the distance eventually becomes infinitesimally small – so small that Achilles runs it instantly. As a result, he catches up with the tortoise, and overtakes him.

At what point does Achilles reach the tortoise? Thanks to the work of

19th Century mathematicians such as Karl Weierstrass, there is a neat rule for this. For any number n between 0 and 1,

$$1 + n + n^2 + n^3 + \ldots = 1/(n-1)$$

In Zeno's problem n=1/10, which means that Achilles will catch the tortoise after 1.11 miles or so.

This result might seem like no more than a historical curiosity – a clever solution to an ancient puzzle. But the idea is still very much relevant today. Rather than using it to study a race between a runner and a reptile, mathematicians are now putting it to work in the fight against diseases.

Since Middle East respiratory syndrome (MERS) was first reported in September 2012, over 400 cases have appeared around the globe. Some outbreaks consist of a single person, infected by an external, but often unknown, source. On other occasions there is a cluster of infected people who had contact with each other.

One way to measure disease transmission is with the reproduction number, denoted R. This is the average number of secondary cases generated by a typical infectious person. If R is greater than one, each infectious person will produce at least one secondary case, and the infection could cause a major epidemic. If R is less than one, the outbreak will eventually fade away.

Even if the infection has so far failed to cause an epidemic, it is still important to know what the reproduction number is. The closer the virus is to that crucial threshold of one, the smaller the hurdle it needs to overcome to spread efficiently.

Using the reproduction number, we can estimate what might happen when a new infection enters a human population. On average, the initial case will generate R secondary cases. These R infections will then generate R more, which means R^2 new cases, and so on.

If R is less than one, this will create a pattern just like Achilles and the tortoise. So if we know what the reproduction number is, we can use the same formula to work out how large an outbreak will be on average:

Average size of an outbreak $= 1 + R + R^2 + R^3 + \ldots = 1/(1-R)$

The problem is that we don't know the reproduction number for MERS. Fortunately, we do know how many cases have been reported in each outbreak. Which means to estimate the reproduction number (assuming that it is below 1), we just have to flip the equation around:

$R = 1 - 1/(\text{average size})$

In the first year of reported MERS cases, disease clusters ranged from a single case to a group of more than 20 people, with an average outbreak size of 2.7 cases. According to the above back-of-the-envelope calculation, the reproduction number could therefore have been around 0.6.

In contrast, there were only two reported clusters of cases in Shanghai during the outbreaks of avian influenza H7N9 in spring 2013. The average outbreak size was therefore 1.1 cases, which gives an estimated reproduction number of 0.1 – much smaller than that for MERS.

Although techniques like these only provide very rough estimates, they give researchers a way to assess disease risk without detailed datasets. Such methods are especially valuable during an outbreak. From avian influenza to MERS, information is at a premium when faced with infections that, much like Zeno, do not give up their secrets easily.

Source: The Conversation